Math 521-721 Qual Sequence Syllabus

Topics Covered

• Group Theory
  – Basic Group Theory: Definition of group, Subgroups, Cayley’s Theorem, Lagrange’s Theorem, Key examples of groups.
  – Quotient Groups: Cosets, Normal Subgroups, Isomorphism Theorems
  – Group Actions: Action of a group on itself, Conjugacy classes, the class equation
  – Sylow Theorems: Proofs using group actions, applications to groups of small order

• Ring Theory
  – Basic Ring theory: units, zero divisors, types of rings (integral domains, fields, etc.) and key examples (polynomial rings, matrix rings)
  – Quotient rings and ideals: Types of ideals (prime ideals, maximal ideals), isomorphism theorems, adjoining elements
  – Factorization in rings. PIDs and UFDs
  – Integral domains, field of fractions, localization

• Field Theory
  – Algebraic and transcendental field elements, adjoining roots of equations
  – Degree of a field extension, the tower law, applications to ruler and compass constructions
  – Galois groups, the fundamental theorem of Galois theory
  – Simple groups, and solvability by radicals
  – Finite fields, transcendental extensions, the transcendence degree

• Module Theory
  – Module basics, free modules and modules given by generators and relations, direct sum, tensor product and Hom.
  – Noetherian rings and Modules, Hilbert basis theorem
  – Structure theory of modules over a PID, Smith normal form, applications of structure theory to abelian groups and linear algebra
  – Tensor, exterior, and symmetric algebra
  – Complexes and cocomplex of modules, exact sequences, diagram chases, homology and cohomology
  – Primary decomposition in Noetherian rings and modules

• Representation Theory of Finite Groups
  – Definitions of representations, modules over the group ring
  – Irreducible representations and Schur’s Lemma, Wedderburn’s Theorem
  – Character theory and the orthogonality relations
  – Introduction to representation theory of the symmetric group

References
Dummit and Foote, Abstract Algebra, 3rd edition. Chapters 1-5, 7-8, 10-14, 15.1-15.2, 17.1, 18