Math 721 – Homework 3

Due Friday, January 31, 2019 at 5pm

Good practice problems (do not turn in solutions):
DF 10.5 Exercises 1, 2, 16, 22, 26

Problem 1 (DF Exercise 3, 4, 5). Let X_1, X_2 be R-modules.

  (a) Show that $X_1 \oplus X_2$ is projective if and only if X_1 and X_2 are projective.

  (b) Show that $X_1 \oplus X_2$ is injective if and only if X_1 and X_2 are injective.

  (c) Show that $X_1 \oplus X_2$ is flat if and only if X_1 and X_2 are flat.

Problem 2 (DF Exercise 14). Let $0 \to A \xrightarrow{\varphi} B \xrightarrow{\psi} C \to 0$ be a sequence of R modules.

  (a) Show that

$$0 \to \mathrm{Hom}_R(X, A) \xrightarrow{\varphi'} \mathrm{Hom}_R(X, B) \xrightarrow{\psi'} \mathrm{Hom}_R(X, C) \to 0$$

is a short exact sequence of abelian groups for all R-modules X if and only if the original sequence is a split short exact sequence. (Hint: consider $X = A$.)

  (b) Show that

$$0 \to \mathrm{Hom}_R(C, X) \xrightarrow{\psi'} \mathrm{Hom}_R(B, X) \xrightarrow{\varphi'} \mathrm{Hom}_R(A, X) \to 0$$

is a short exact sequence of abelian groups for all R-modules X if and only if the original sequence is a split short exact sequence.