Math 721 – Homework 10

Due Friday, April 17 at 5pm

Good practice problems (do not turn in solutions):
DF 18.1: 1, 2, 3, 4, 5, 6, 8, 14, 18
DF 18.2: 14, 15, 16

Problem 1 (DF 18.1.15 & 16)**.** Let G be a finite abelian group. This exercise concerns 1-dimensional complex representations of G. (Here "complex" means "over $\mathbb{C}$".)

(a) If G is cyclic, exhibit all 1-dimensional complex representations of G. Make sure to decide which are inequivalent.

(b) For arbitrary finite abelian group G, exhibit all 1-dimensional complex representations of G and decide which are inequivalent.

(c) Conclude that the number of inequivalent 1-dimensional complex representations of G equals $|G|$.

Problem 2 (DF 18.2.12)**.** Let F be a field, $f(x) \in F[x]$, and $R = F[x]/f(x)$.

(a) Find necessary and sufficient conditions on the factorization of $f(x)$ in $F[x]$ so that R is a semisimple ring.

(b) When R is semisimple, describe its Wedderburn decomposition.